EXPERIMENT 8: ZEEMAN EFFECT

Related Topics
Bohr’s atomic model, quantisation of energy levels, electron spin, Bohr’s magneton, interference of electromagnetic waves, Fabry-Perot interferometer

Principle
The “Zeeman effect” is the splitting up of the central spectral lines of atoms within a magnetic field. The simplest is the splitting up of one spectral line into three components called the “normal Zeeman effect”. The normal Zeeman effect is studied using a cadmium spectral lamp as a specimen. The cadmium lamp is submitted to different magnetic flux densities and the splitting up of the red cadmium line (643.8 nm) is investigated using a Fabry-Perot interferometer. The evaluation of the results leads to a fairly precise value for Bohr’s magneton.

Tasks
1. Using the Fabry-Perot interferometer and a selfmade telescope the splitting up of the central line into two s-lines is measured in wave numbers as a function of the magnetic flux density.
2. From the results of point 1. a value for Bohr’s magneton is evaluated.
3. The light emitted within the direction of the magnetic field is qualitatively investigated.

Theory and Evaluation
As early as 1862, Faraday investigated whether the spectrum of coloured flames changes because of the effect of a magnetic field, but without success. It was not until 1885 that the Belgian Fievez was able to demonstrate an effect, but it was forgotten and only rediscovered 11 years later by the Dutchman Zeeman, who studied it together with Lorentz. This experiment, which was of importance to the development of the theory of the atomic shell, can now be carried out with modern equipment in the students’ experiment laboratory. The splitting of the Cd-spectral line $\lambda = 643.8$ nm into three lines, the so-called Lorentz triplets, occurs since the Cd-atom represents a singlet system of total spin $S = 0$. In the absence of a magnetic field there is only one possible $D \rightarrow P$ transition of 643.8 nm. In the presence of a magnetic field the associated energy levels split into $2L + 1$ components. Radiating transitions between these components are possible, provided that the selection rules

$$\Delta M_L = +1; \quad \Delta M_L = 0; \quad \Delta M_L = -1;$$

are taken into account. In this case, therefore, there are a total of nine permitted transitions, only three of which ever have the same energy and hence the same wavelength. Therefore, only three lines will be visible.
Fig. 1: Splitting up of the components in the magnetic field and permitted transitions.

The first group where $\Delta M_L = -1$ gives a $\sigma$-line whose light is polarized vertically to the magnetic field. The middle group $\Delta M_L = 0$ gives a $\pi$-line whose light is polarized parallel to the direction of the field. The last group where $\Delta M_L = +1$ gives a $\sigma$-line whose light is again polarized vertically to the magnetic field.

In the absence of the analyser all three lines can be seen simultaneously. Each ring which was observed in the absence of a magnetic field is split into three rings when a magnetic field is applied. Inserting the analyser the two $\sigma$-lines can be observed exclusively if the analyser is in the vertical position, while only the $\pi$-line appears if the analyser is turned into its horizontal position (transverse Zeeman effect). Turning the electromagnet by 90° the light coming from the spectral lamp parallel to the direction of the field can also be studied since the pole-shoes have been drilled. It can be shown that this light is circular polarized light. Whatever the position of the analyser may be, each of the rings seen without a magnetic field is now permanently split into two rings in the presence of a magnetic field (longitudinal Zeeman effect).
Fig. 2: Longitudinal and transverse Zeeman effect.

Turning the electromagnet back for the observation of the two $\sigma$-lines of the transverse Zeeman effect it is easy to see that the size of the splitting increases with increasing magnetic field strength. For a quantitative measurement of this splitting in terms of number of wavelengths, a Fabry-Perot interferometer is used, the functioning of which may briefly be explained.

Fig. 3: Reflected and transmitted rays at the parallel surfaces (1) and (2) of the etalon. The etalon spacing is $t$.

The Fabry-Perot étalon has a resolution of approximately 300000. That means that a wavelength change of approximately 0.002 nm can still be detected. The étalon consists of two parallel flat glass plates coated on the inner surface with a partially transmitting metallic layer. Let us consider the two partially transmitting surfaces (1) and (2) in Fig. 3 separated by a distance $t$. An incoming ray making an angle $\theta$ with the normal to the plates will be split into the rays AB, CD,
EF, etc. the path difference between the wave fronts of two adjacent rays (for example, AB and CD) is

$$\delta = BC + CK$$

Where, obviously, BK is normal to CD. With

$$CK = BC \cos 2\theta \text{ and } BC \cos \theta = t$$

we obtain

$$\delta = BCK = BC(1 + \cos 2\theta)$$

$$= 2 BC \cos^2 \theta$$

$$= 2 t \cos \theta$$

and for a constructive interference to occur

$$n\lambda = 2 t \cos \theta$$

where n is an integer. If the refractive index of the medium between the plates is $\mu \neq 1$, the equation still has to be modified in the following way:

$$n\lambda = 2 \mu t \cos \theta$$

Equation (1) is the basic interferometer equation. Let the parallel rays B, D, F, etc. be brought to a focus by the use of a lens of focal length $f$ as shown in Fig. 4.

![Fig. 4: Focusing of the light emerging from a Farby-Perot étalon. Light entering the étalon at an angle $\theta$ is focused onto a ring of radius $r = f \theta$ where $f$ is the focal length of the lens.](image)

Then, when $\theta$ fulfills equation (1), bright rings will appear in the focal plane, their radius being given by

$$r_n = f \tan \theta_n = f \theta_n$$

for small values $\theta_n$, e.g. rays nearly parallel to the optical axis.
\[ n = \frac{2 \mu t}{\lambda} \cos \theta_n = n_0 \cos \theta_0 \]

\[ = n_0 \left( 1 - 2 \sin^2 \frac{\theta_n}{2} \right) \]

with

\[ n_0 = \frac{2 \mu t}{\lambda} \]

we finally obtain

\[ n = n_0 \left( 1 - \frac{\theta_n^2}{2} \right) \]

\[ \theta_n = \sqrt{\frac{2(n_0 - n)}{n_0}} \]

If \( \theta_n \) is to correspond to a bright fringe, \( n \) must be an integer. However, \( n_0 \), which gives the interference at the center (\( \cos \theta = 1 \) or \( \theta = 0 \)), is in general not an integer. If \( n_1 \) is the interference order of the first ring, clearly \( n_1 < n_0 \) since \( n_1 = n_0 \cos \theta_{n1} \). We then let

\[ n_1 = n_0 - \varepsilon ; \quad 0 < \varepsilon < 1 \]

where \( n_1 \) is the closest integer to \( n_0 \) (smaller than \( N_0 \)). Thus, we have in general for the \( p \)-th ring of the pattern, as measured from the center out,

\[ n_p = (n_0 - \varepsilon) - (p - 1) \]

\[ r_p^2 = \frac{2f^2}{n_0} \cdot \sqrt{(p-1) + \varepsilon} \]

we note that the difference between the squares of the radii of adjacent rings is a constant:

\[ r_{p+1}^2 - r_p^2 = \frac{2f^2}{n_0} \]

\( \varepsilon \) can be determined graphically plotting \( r_p^2 \) versus \( p \) and extrapolating to \( r_p^2 = 0 \).

Now, if there are two components of a spectral line (splitting of one central line into two components) with wavelengths \( \lambda_a \) and \( \lambda_b \), which are very close to one another, they will have fractional orders at the center \( \varepsilon_a \) and \( \varepsilon_b \):
\[ \epsilon_a = \frac{2 \mu t}{\lambda_a} - n_{1,a} = 2 \mu t \bar{\nu}_a - n_{1,a} \]

\[ \epsilon_b = \frac{2 \mu t}{\lambda_b} - n_{1,b} = 2 \mu t \bar{\nu}_b - n_{1,b} \]

where \( n_{1,a}, n_{1,b} \) is the interference order of the first ring. Hence, if the rings do not overlap by a whole order \( n_{1,a} = n_{1,b} \) and the difference in wave numbers between the two components is simply

\[ \Delta \bar{\nu} = \bar{\nu}_a - \bar{\nu}_b = \frac{\epsilon_a - \epsilon_b}{2 \mu t} \]

\[ \frac{r^2_{p+1,a}}{r^2_{p+1} - r^2_p} - p = \epsilon \]

yields

\[ \frac{r^2_{p+1,a}}{r^2_{p+1,a} - r^2_{p,a}} - p = \epsilon_a \]

and

\[ \frac{r^2_{p+1,b}}{r^2_{p+1,b} - r^2_{p,b}} - p = \epsilon_b \]

\[ \Delta \bar{\nu} = \frac{1}{2 \mu t} \left( \frac{r^2_{p+1,a}}{r^2_{p+1,a} - r^2_{p,a}} - \frac{r^2_{p+1,b}}{r^2_{p+1,b} - r^2_{p,b}} \right), \quad \Delta \bar{\nu} = \frac{1}{2 t \Delta} \]

The étalon spacing is \( t = 3 \cdot 10^{-3} \) [m].
**Equipment**

Fabry-Perot interferometer, Cadmium lamp, Electromagnets, Digital Multimeters

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**Fig.5:** Experimental set-up for Zeeman Effect.

**Set-up and Procedure**

The electromagnet is put on the rotating table for heavy loads and mounted with the two pole-shoes with holes in such a way that a gap of 9 mm remains for the Cd-lamp. The poleshoes have to be well tightened in such a way that they can't move later on when the magnetic flux is established. The Cd-lamp is inserted into the gap without touching the poleshoes and connected to the power supply for spectral lamps. The coils of the electromagnet are connected in parallel and via an ammeter connected to the variable power supply of up to 20 VDC, 12 A. A capacitor of 22000 μF is in parallel to the power output to smoothen the DC-voltage.

The optical bench for investigation of the line splitting carries the following elements (their approximate position in cm is given in brackets):
The iris diaphragm is eliminated for initial adjustment and for the observation of the longitudinal Zeeman effect. During observation of the transverse Zeeman effect the iris diaphragm is illuminated by the Cd-lamp and acts as such as the light source. The lens $L_1$ and a lens of $f = 100$ mm, incorporated in the étalon, create a nearly parallel light beam which the Fabry-Perot étalon needs for a proper interference pattern. The étalon contains an interchangeable colour filter which filters out the red cadmium line of 643.8 nm. The lens $L_2$ produces and interference pattern of rings within the plane of the screen with a scale mounted on a slide mount which can latterally be displaced with a precision of $1/100$ of a millimeter. The ring system is observed through $L_3$ and the ring diameters can be measured, for instance, by systematic displacement of the slash representing the “0” of the scale. The readings should be done in a completely darkened room using a flashlight.

The initial adjustment is done in the following way:
The rotating table with electromagnet, pole-shoes and Cd-lamp already mounted is brought to a height of about 16 cm above table using the supporting blocks. By means of the spirit level, the electromagnet is adjusted perfectly horizontal. The optical bench with all elements (except is iris diaphragm) mounted, is then moved closer to the electromagnet in such a way that one of the outlet holes of the pole-shoes coincides with the previous position of the iris diaphragm. $L_1$ is then adjusted so that the outlet hole is within the focus of it. All other optical elements of Fig. 6 are subsequently realigned with respect to their height correspondingly. The current of the coils is set for some time to 8 [A] (increase in light intensity!) and the ring interference pattern in axial direction is observed through $L_3$. The pattern must be centered and sharp which is eventually achieved by a last, slight movement of the étalon (to the right or to the left) and by displacement of $L_2$ (vertically and horizontally). Finally the screen with scale is shifted in a way that the slash representing the “0” of the scale is clearly seen coinciding, for instance, with the
center of the fairly bright inner ring. The scale itself must be able to move horizontally along the diameter of the ring pattern.

The electromagnet is now turned by 90°, the iris diaphragm is inserted and the analyser turned until the $\pi$-line (expl. follows) disappears completely and the two $\sigma$-lines appear clearly visible.

Remark: For later evaluations the calibration curve of the magnetic flux density versus the coil current has to be traced previously. This can be done if a teslameter is available. Otherwise the results of Fig. 7 must be used. The curve of Fig. 3 was traced by measuring the flux density in the center of the gap in the absence of the Cd-lamp. For the evaluations these center-values were increased by 3.5% to account for the non-uniform flux distribution within the gap.

Fig. 6: Arrangement of optical components.

Fig. 7: Magnetic flux density $B$ in the center of gap without the Cd-lamp (gap width: 9 mm) as a function of the coil current.